DIAS Delhi Institute for Administrative Services
India's Leading Institute for Civil Services Examination

ALL INDIA TEST SERIES CSE-2023

PH TS-2401040

Candidate 's Information

1. NAME:- VIBHOR BHARDWAJ
2. UPSC ROLL NO:- 0833456
3. MOBILE NO:- [redacted]
4. SUBJECT:- SOLID STATE PHYSICS
5. DATE:- 21-07-2024

FOR OFFICE USE ONLY:-

Q.NO	MARKS
1.	26
2.	33
3.	30
4.	
5.	$31\frac{1}{2}$
6.	25
7.	
8.	

TOTAL MARKS	$\frac{146}{250}$

Very Good.

EXAMINER SIGNATURE

INVIGILATOR SIGNATURE

18 Pusa Road Karol Bagh New Delhi-110005 Ph:01140079000, 9350934622 e-mail diasindia123@gmail.com

Question - 1

Ans 1(a) Critical field is magnetic field beyond which the superconductor behaves like normal conductor.

Given $H_{c1} = 7.616\ MAm^{-1}$ at $T_1 = 6K$
$H_{c2} = 4.284\ MAm^{-1}$ at $T_2 = 8K$

(i) $$H_c = H_c(0)\left(1 - \frac{T^2}{T_c^2}\right) \longrightarrow ①$$

$H_c(0)$ → critical field at 0K.

$$\frac{H_{c1}}{H_{c2}} = \frac{(1 - T_1^2/T_c^2)}{(1 - T_2^2/T_c^2)}$$

$$\frac{7.616}{4.284} = \frac{1 - 36/T_c^2}{(1 - 64/T_c^2)}$$

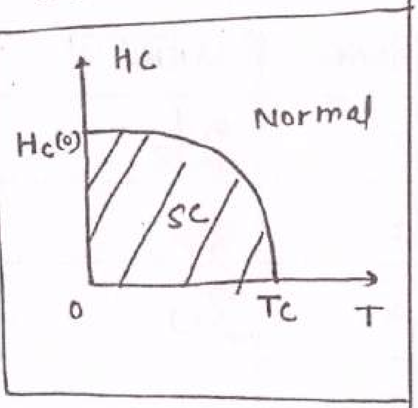
$$1.78 = \frac{T_c^2 - 36}{T_c^2 - 64}$$

$$1.78 T_c^2 - 113.92 = T_c^2 - 36.$$

$$T_c^2 = 99.89$$

$$T_c = 9.99K$$

Transition temp. $\boxed{T_c \sim 10K}$



Putting T_c value in eqn-(1)

$$\frac{H_{c1}}{\left(1-\frac{T_1^2}{T_c^2}\right)} = H_c(0)$$

$$\frac{7.616}{\left(1-\frac{36}{100}\right)} = H_c(0)$$

$$\boxed{H_c(0) = 11.9 \text{ MA m}^{-1}}$$

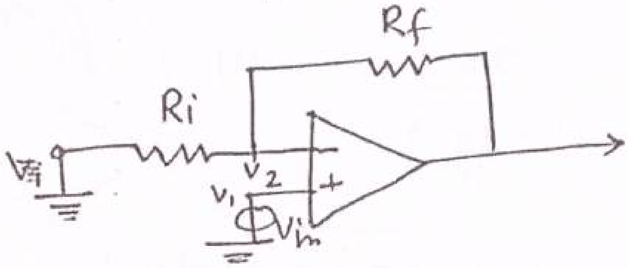
$6\frac{1}{2}$ / 10

Beyond critical temp., the superconductor is normal conductor.

Ans 1(b) Non-inverting OPAMP : $R_i = 1K$

$R_f = 99K$

$A_{op} = 500{,}000$



(i) $\beta = \dfrac{R_i}{R_i + R_F} = \dfrac{1K}{1K + 99K} = \dfrac{1}{100} = \underline{0.01}$.

(ii) Loop gain $= \beta A_{OL} = 5 \times 10^5 \times 10^{-2}$.

$= \underline{5 \times 10^3}$.

(iii) Exact closed loop gain $= \dfrac{A_{OL}}{1 + \beta A_{OL}}$

A_{exact}

$= \dfrac{5 \times 10^5}{1 + 5 \times 10^3}$

$\cong \underline{99.98}$

$\frac{1\frac{1}{2}}{10}$

(iv) $A_{exact} = \dfrac{A_{OL}}{1 + \beta A_{OL}}$

$= \dfrac{A_{OL}}{A_{OL}\left(\frac{1}{A_{OL}} + \beta\right)} = \dfrac{1}{\beta + \frac{1}{A_{OL}}}$

at $A_{OL} = \infty \Rightarrow A_{exact} = \dfrac{1}{\beta} = \underline{100}$ Ans

Ans I(C) AC Josephson effect: when two superconductors separated by a thin insulating layer are connected to a battery (D.C.), there flows a current through the insulating layer. This is called AC Josephson effect.

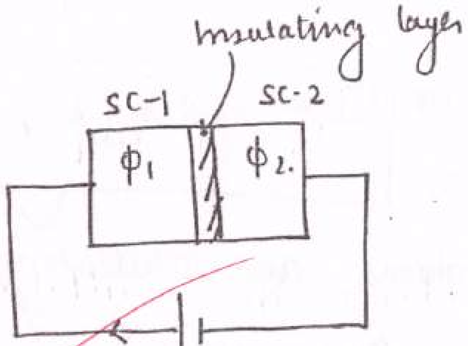


Fig. Josephson effect (a.c.)

Let there be two superconductors with superelectron density (n_1 and n_2) and phases ϕ_1, ϕ_2 respectively.

Schrödinger time dependent equation:

$$i\hbar \frac{\partial \psi_1}{\partial t} = eV\psi_1 + \hbar T \psi_2 \quad \text{(coupling)} \longrightarrow (1)$$

$$i\hbar \frac{\partial \psi_2}{\partial t} = -eV\psi_2 + \hbar T \psi_1 \longrightarrow (2).$$

putting $\psi_1 = \sqrt{n_1}e^{i\theta_1}$ and $\psi_2 = \sqrt{n_2}e^{i\theta_2}$.

and solving we get:

$$\frac{d}{dt}(\theta_2 - \theta_1) = -\frac{2eV}{\hbar} \quad \& \quad \frac{dn_1}{dt} = -\frac{dn_2}{dt}.$$

$$\text{or} \quad \frac{d\delta}{dt} = -\frac{2eV}{\hbar}.$$

Now current density $J = n_2 \frac{dn_2}{dt}$

$$J = J_0 \sin\left[\delta(0) - \frac{2eV}{\hbar}t\right]$$

$$\text{or} \quad \boxed{I = I_0 \sin\left[\delta(0) - \frac{2eV}{\hbar}t\right]}$$

which shows an oscillatory current with phase $\left(\delta(0) - \frac{2eV}{\hbar}t\right)$

$\frac{6\frac{1}{2}}{10}$

Ans 1(d) Intrinsic semiconductor has no doping or impurity.

Given: Resistivity of semiconductor $\rho_1 = 4.5\ \Omega\text{-m}$
at $T_1 = 20^\circ C = 293\ K$

and $\rho_2 = 2\ \Omega\text{-m}$ at $T_2 = 32^\circ C$
$= 305\ K$.

To find: Energy gap $E_g = ?$

6 1/2 / 10

conductivity $\sigma_1 = \frac{1}{\rho_1} = 2.2 \times 10^{-1} (\Omega\text{-m})^{-1}$

$\sigma_2 = 0.5\ (\Omega\text{-m})^{-1}$.

For intrinsic semiconductor:

$\sigma = n_i e \mu$ where $n \rightarrow e^-$ concentration
$e \rightarrow$ charge of e^-
$\mu \rightarrow$ mobility of carrier

&

$$n_i = 2 \cdot \left(\frac{2\pi \cancel{m^*} KT}{h^2}\right)^{3/2} \cdot e^{-E_g/2KT} (m_e^* m_p^*)^{3/4}.$$

so $n_i \propto T^{3/2} \cdot e^{-E_g/2KT}$

so $$\frac{\sigma_1}{\sigma_2} = \left(\frac{T_1}{T_2}\right)^{3/2} \frac{e^{-E_g/2KT_1}}{e^{-E_g/2KT_2}} = \left(\frac{T_1}{T_2}\right)^{3/2} e^{-E_g/2K\left(\frac{1}{T_1} - \frac{1}{T_2}\right)}$$

or $$\ln \frac{\sigma_1}{\sigma_2} = \frac{3}{2} \ln \frac{T_1}{T_2} - \frac{E_g}{2K}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

putting values:

$$\frac{E_g}{2K}\left(\frac{1}{T_1}-\frac{1}{T_2}\right) = \ln\frac{\sigma_2}{\sigma_1} + \frac{3}{2}\ln\frac{T_1}{T_2}$$

$$\frac{E_g}{2K}\left[\frac{1}{293}-\frac{1}{305}\right] = \ln\frac{0.5}{0.22} + \frac{3}{2}\ln\frac{293}{305}$$

$$E_g = (0.82 - 0.06) \times \frac{293\times305}{12} \times 2\times 1.38\times10^{-23}$$

$$= \frac{1.54\times10^{-19}\,J}{1.6\times10^{-19}}$$

$$\boxed{E_g = 0.9625\ eV}$$

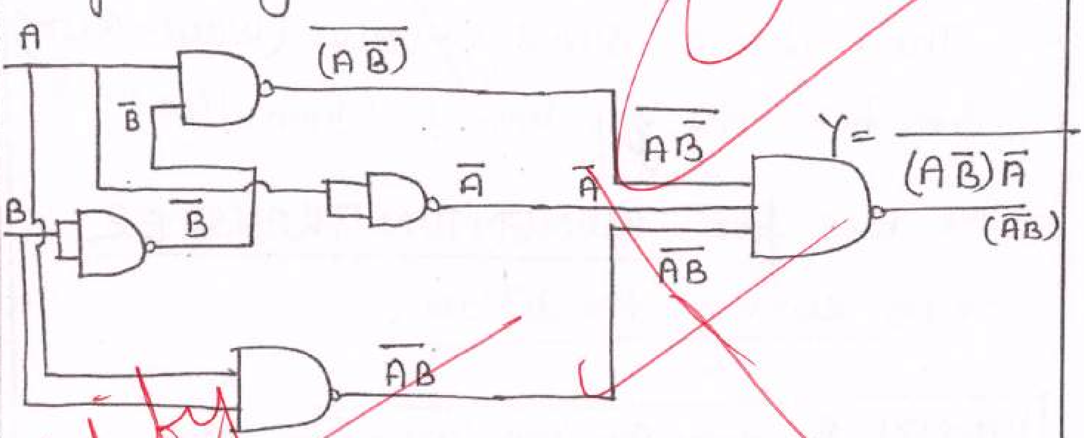
Ans 1(e) Boolean expression $Y = \overline{A\overline{B} + A + AB}$

$$\boxed{Y = \overline{(A\overline{B})} \cdot \overline{A} \cdot \overline{AB}}$$

using De-Morgan's theorem of

$$\overline{A+B} = \overline{A} \cdot \overline{B}$$

using NAND gates:



Simplify

(NAND gate logical circuit of given expression)

NAND gate is a $\boxed{\text{universal gate}}$ as all other gates can be realised using this.

Ans 2

Ans 2(a) Direct Band gap semiconductor: The semiconductors having coincidence of top of valence band with bottom of conduction band are called direct band gap semiconductor.

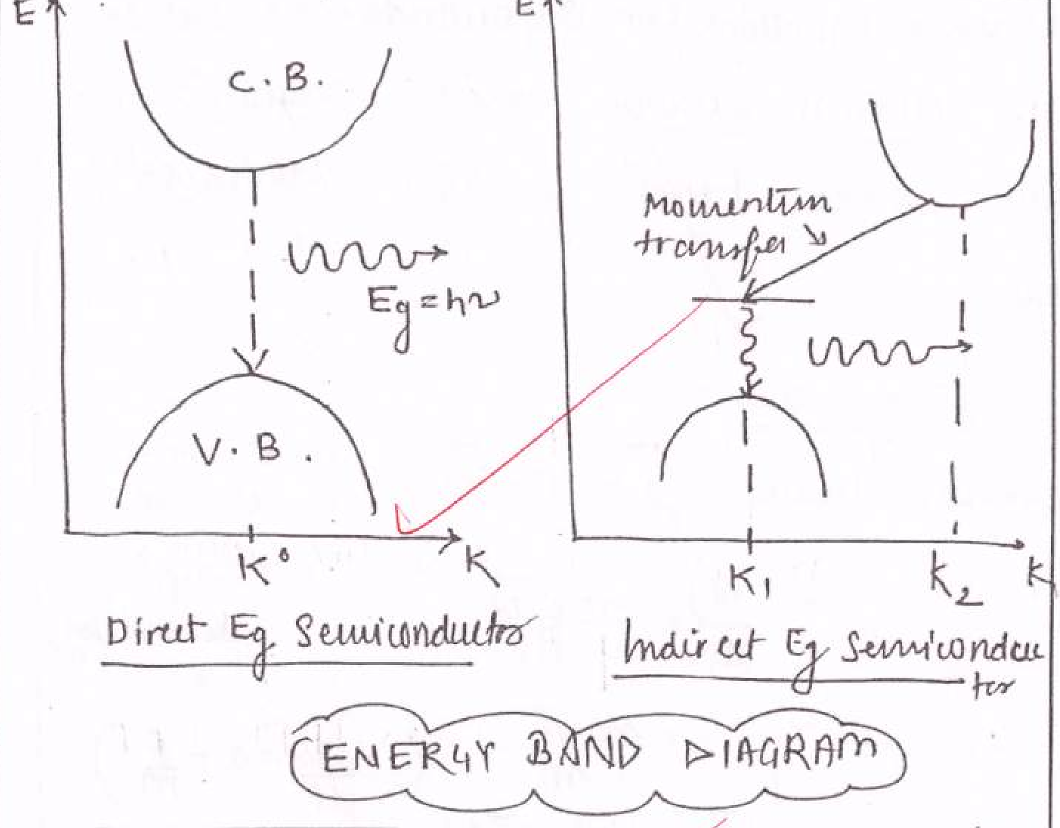
→ These have same K-value (wave-vector) for both the ~~gap~~ bands. (transition)

→ No need for MOMENTUM TRANSFER vector during transition.

INDIRECT Bandgap Semiconductor — The conduction band bottom does not coincide with the ~~bo~~ top of valence band.

→ Need to have a momentum transfer during transition.

→ Have different K-values for ~~b~~ conduction and valence band extremum.



In SOLAR CELLS direct bandgap semiconductors are more suitable as the absorption of solar radiation photon and subsequent e⁻-h pair generation can take place without any PHONON or crystal lattice involvement or need to transfer a momentum i.e. more efficiency is ensured.

Eg. Silicon

10/15

Ans 2(b) LONDON equations for superconductors relate the current density with magnetic field and hence predicts Meissner effect.

(i) FIRST equation –

current density $\vec{j} = ne\vec{v}_d$

$n \rightarrow e^-$ conc.
$e \rightarrow$ charge
$v_d \rightarrow$ drift velocity of e^-

or $\frac{d\vec{j}}{dt} = ne\frac{d\vec{v}_d}{dt}$

$= ne\frac{\vec{F}_{el}}{m} \quad \left(\because \frac{dV_d}{dt} = a = \frac{F}{m}\right)$

$\frac{d\vec{j}}{dt} = ne\frac{(e\vec{E})}{m} = \frac{ne^2\vec{E}}{m}$

$$\boxed{\frac{d\vec{j}}{dt} = \frac{ne^2}{m}\vec{E}} \longrightarrow (1)$$

(ii) Taking curl of eqn – (1)

$\frac{d}{dt}(\vec{\nabla}\times\vec{j}) = \frac{ne^2}{m}(\vec{\nabla}\times\vec{E})$

$= -\frac{ne^2}{m}\frac{d\vec{B}}{dt} \quad (\because$ Maxwell eqn$)$

$\Rightarrow \frac{d}{dt}\left(\vec{\nabla}\times\vec{j} + \frac{ne^2}{m}\vec{B}\right) = 0$

or $$\boxed{\vec{\nabla}\times\vec{j} + \frac{ne^2}{m}\vec{B} = 0} \longrightarrow (2)$$ 2nd equation

$\vec{B} = \vec{\nabla}\times\vec{A}$

$\vec{J} = \frac{-ne^2}{m}\vec{A}$ (2)

Again taking curl of the equation (2)(4):

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{B} = \mu_0 \vec{J})$$ (electrostatic one)

$$-\nabla^2 B = \mu_0 \, \vec{\nabla} \times \vec{J} \quad (\because \vec{\nabla} \cdot \vec{B} = 0)$$

putting $\vec{\nabla} \times \vec{J} = -\frac{ne^2}{m} B$.

$$-\nabla^2 B = -\mu_0 \frac{ne^2}{m} B$$

$$\boxed{\nabla^2 B = \frac{1}{\lambda_L^2} B}$$

$\lambda_L = \sqrt{\frac{m}{\mu_0 n e^2}}$ = London penetration depth.

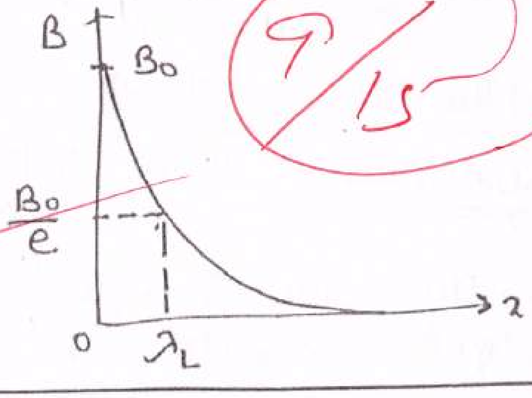
The above equation shows that the solution is not uniform in space. But it can be only present in layer λ_L.

On the other hand $\frac{\partial \vec{B}}{\partial t} = 0$ requires $\vec{B}$ is constant.

It is possible only if $|\vec{B}| = 0$.

which predicts the Meissner's effect.

(ii) The magnetic flux inside the superconductor is zero.



9/15

The flux penetrates only upto few unit of length and becomes $\frac{1}{e}$ of maximum value at λ_L.

$$\boxed{\vec{B}(x) = B_0 e^{-x/\lambda_L}}$$

Ans 2(c) **Debye's theory of Lattice Heat Capacity** —

Debye modified Einstein's theory and put forth the assumptions —

(1) In a solid, atoms behave like coupled Harmonic oscillator.

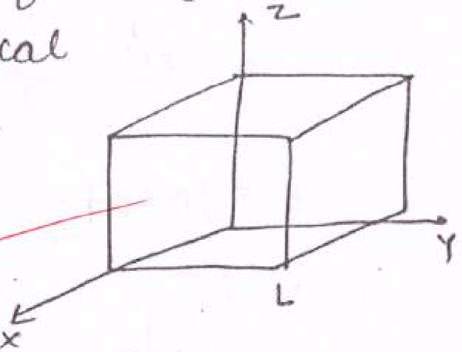
(2) There is a possibility of a number of frequency modes present with boundary limit of Debye frequency ν_D.

(3) The solid behaves like an elastic continuum with range of frequencies.

(4) There are 3 acoustic modes of vibrations — (i) One Longitudinal and (ii) 2 transverse

The density of modes for Debye-model:

Let there be solid of cubical shape with length 'L'.



we know that for wavefunction $\psi = e^{i}$

periodic boundary conditions yield:

$$e^{i(k_x x + k_y y + k_z z)} = e^{i[(x+L)k_x + (y+L)k_y + (z+L)k_z]}$$

giving $e^{iL(k_x+k_y+k_z)} = 1$

or $\boxed{k_x + k_y + k_z = \frac{2\pi}{L}}$

The density of modes $g(k)\,dk = \frac{1}{8} \times \frac{4\pi k^2 dk}{(\pi/L)^3}$

$$= \frac{V k^2 dk}{2\pi^2}$$

14/20

$\because$ velocity $v = \frac{\omega}{k} \Rightarrow k = \frac{\omega}{v} \Rightarrow dk = \frac{d\omega}{v}$

so, $\because$ $\nu = \frac{\omega}{2\pi} \Rightarrow d\nu = \frac{d\omega}{2\pi}$.

$g(\omega)\,d\omega = \frac{V}{2\pi^2} \cdot \frac{\omega^2 d\omega}{v^3} = \frac{V}{2\pi^2} \cdot \frac{1}{v^3} \cdot 4\pi^2 \nu^2 d\nu\,(2\pi)$

$$\boxed{g(\nu)\,d\nu = \frac{4\pi V \nu^2 d\nu}{v^3}}$$

or $\bar{E} = \frac{9Nh}{\nu_D^3} \cdot \int_0^{\nu_D} \nu^2 d\nu \left(\frac{KT}{h}\right)$

$= \frac{9Nh}{\nu_D^3} \times \frac{\nu_D^3}{3} \times \frac{KT}{h} = 3Nk_BT$

$E = 3Nk_BT$

or $C_V = \left.\frac{\partial E}{\partial t}\right|_V = 3Nk_B = 3R = 5.96$ Cal/Mol-K

$$\boxed{C_V = 5.96 \text{ cal/mol-k}}$$ (constant)

(ii) for low temp; $\frac{h\nu}{KT} \gg 1$ or $x \gg 1$

$\frac{h\nu_D}{kT} \to \infty$.

so $\int_0^{\infty} \frac{\nu^3 d\nu}{e^{h\nu/KT}-1} = \int_0^{\infty} \left(\frac{KT}{h}\right)^3 \cdot \frac{x^3}{e^x-1} \cdot \left(\frac{KT}{h}\right) \cdot dx$

$= \left(\frac{KT}{h}\right)^4 \int_0^{\infty} \frac{x^3}{e^x-1} dx = \frac{\pi^4}{15} \cdot \left(\frac{KT}{h}\right)^4$

putting in eqn-(ii)

$E = \frac{9Nh}{\nu_D^3} \cdot \frac{\pi^4}{15} \left(\frac{KT}{h}\right)^4 = \frac{3}{5}\pi^4 \frac{K^4 N}{h^3 \nu_D^3} \cdot T^4$

$C_V = \left.\frac{\partial E}{\partial T}\right|_V = \frac{12}{5}\pi^4 \left(\frac{K}{h\nu_D}\right)^3 kT^3 . N$

$$\boxed{C_V = \frac{12}{5}\pi^4 kN\left(\frac{T}{\theta_D}\right)^3} = \frac{12}{5}\pi^4 R\left(\frac{T}{\theta_D}\right)^3$$

$\because \quad \frac{1}{v^3} = \left(\frac{1}{v_l^3} + \frac{2}{v_t^3}\right)$ where v_l = velocity for longitudinal mod

v_t = velocity for transverse mod.

so $\left[g(\nu)d\nu = 4\pi V\left(\frac{1}{v_l^3} + \frac{2}{v_t^3}\right) \cdot \nu^2 d\nu\right]$

for Total modes to be 3N.

$$\int_0^{\nu_D} g(\nu)\,d\nu = 3N \quad \text{or} \quad 4\pi V\left(\frac{1}{v_l^3} + \frac{2}{v_t^3}\right)\frac{\nu_D^3}{3} = 3N$$

$$\text{or} \quad \left(\frac{1}{v_l^3} + \frac{2}{v_t^3}\right) = \frac{9N}{4\pi V\nu_D^3} \longrightarrow \text{(I)}$$

Now The energy average $= \int_0^{\nu_D} E\,g(\nu)\,d\nu$

$$= \int_0^{\nu_D} \frac{h\nu}{(e^{h\nu/KT}-1)} \cdot \frac{4\pi}{\cancel{3}} \cdot \nu^2 \cdot \frac{9N}{4\pi\nu_D^3}\,d\nu$$

$$\bar{E} = \frac{9Nh}{\nu_D^3}\int_0^{\nu_D} \frac{\nu^3 d\nu}{e^{h\nu/KT}-1} \longrightarrow \text{(II)}$$

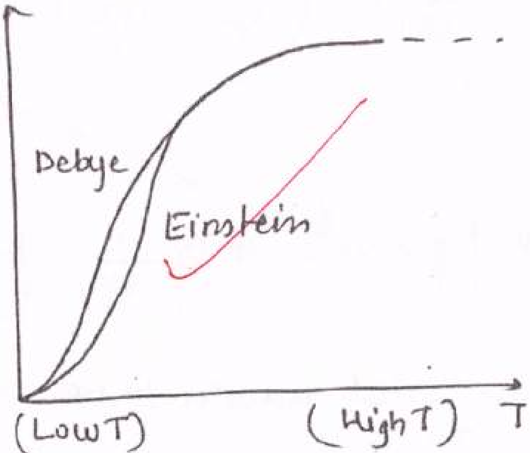
let $\frac{h\nu}{KT} = x$ and $\frac{h\nu_D}{KT} = x_m$.

so,

(i) for High temp. $\frac{h\nu}{KT} \ll 1$ or $x \ll 1$.

so $e^{h\nu/KT} - 1 \cong \frac{h\nu}{KT}$.

or $\boxed{C_v \propto T^3}$



Difference from Einstein's theory -

① Einstein considered atoms to be INDEPENDENT Harmonic oscillator.

While Debye expressed them as coupled.

② Einstein took single frequency "ω_0" with which the whole crystal vibrated while Debye took a range of frequencies.

③ Debye model explained "C_v" behaviour at low 'T' while Einstein model couldn't do it.

However there are challenges for Debye model such as -

(i) Not for polyatomic.

(ii) ν_D not constant for v_L and v_T modes.

(iii) Only for short frequencies (long wavelength)

Question: 3

Ans 3(a) Debye Temp. of diamond $\theta_D = 2000\,K$.

Density $\rho = 3500\ K/m^3$

Atomic mass $= 12$ amu

Interatomic Spacing $= 1.54\,\mathring{A}$.

(i) Mean velocity of sound : v

$$\theta_D = \frac{h\nu_D}{k_B} \Rightarrow \nu_D = \frac{k_B\theta_D}{h} = \frac{1.38\times10^{-23}\times 2000}{6.6\times10^{-34}}$$

$$\boxed{\nu_D = 4.18\times10^{13}\ Hz}$$

$\because$

$$\int_0^{\nu_D} g(\nu)\,d\nu = 3N$$

or

$$\left(\frac{1}{V_l^3}+\frac{2}{V_t^3}\right) = \frac{9N}{4\pi\nu_D^3 V}$$ (from ques: 2(c) eq^n-①)

for average velocity, $V_l = V_t = v$.

so

$$\frac{3}{v^3} = \frac{9N}{4\pi\nu_D^3 V}$$

$$v^3 = \frac{4\pi\nu_D^3 V}{3N}$$

$$v = \left(\frac{4\pi V}{3N}\right)^{\frac{1}{3}}\nu_D$$

Here $V = \frac{\text{Mass}}{\text{Density}} = \frac{M}{\rho N_A}$ so $v = \left(\frac{4\pi M}{3\rho N_A}\right)^{\frac{1}{3}}\nu_D$.

So $v = \left[\dfrac{4 \times 3.14 \times 12}{6.0 \times 10^{23} \times 10^{3} \times 3 \times 3500}\right]^{1/3} \cdot 4.18 \times 10^{13}$

$= 2.08 \times 10^{-10} \times 4.18 \times 10^{13}$

$\boxed{v = 1.2 \times 10^{4} \text{ m/sec}}$

(ii) Frequency of dominant mode:

Dominant wavelength $\lambda \cong \left(\dfrac{\theta_D}{T}\right)$ (Lattice constant)

For room temp $T = 300k$.

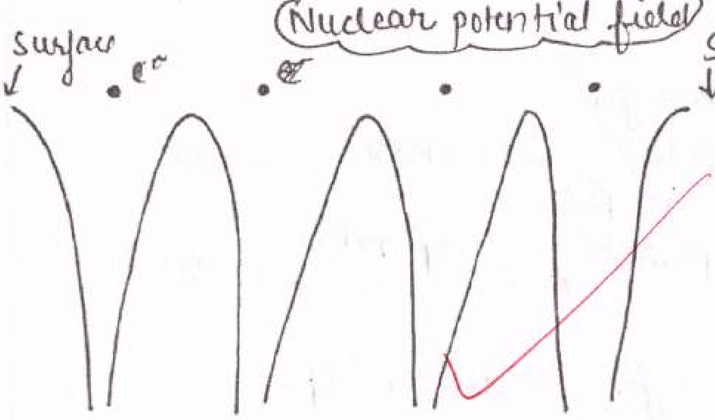
$\nu = \dfrac{v}{\lambda} = \dfrac{1.2 \times 10^{4} \times 300}{2000 \times 1.54 \times 10^{-10}}$

$= \dfrac{1.8}{1.54} \times 10^{3} \times 10^{10}$

$\boxed{\nu = 1.16 \times 10^{13} \text{ Hz}}$ Ans

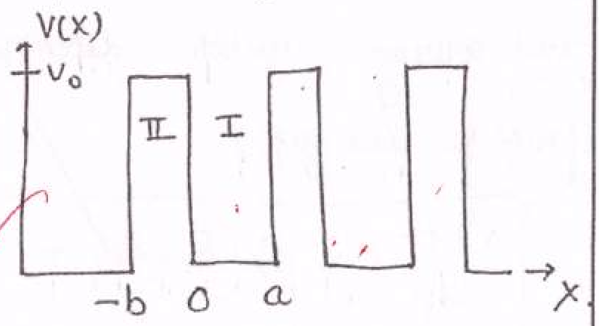
10/15

Ans 3(b) Motion of an electron in 1-D periodic potential:



Lets consider a periodic potential array with period (a+b) and potential form be square potential.

$V(x) = 0 \quad 0 \le x \le a$
$\quad\quad V_0 \quad -b \le x \le 0.$



with $E < V_0$

Schrödinger equations:

$$\frac{d^2\psi}{dx^2} + \alpha^2\psi = 0 \quad 0 \le x \le a \quad , \quad \alpha = \sqrt{\frac{2mE}{\hbar^2}}$$

$$\frac{d^2\psi}{dx^2} - \beta^2\psi = 0 \quad -b \le x \le a \quad\quad \beta = \sqrt{\frac{2m(V_0-E)}{\hbar^2}}$$

let $\psi = u_k e^{iKx}$ with $u_k(x) = u_k(x+L)$ is Bloch function

putting in both eq^ns —

$$\frac{d^2u_1}{dx^2} + 2iK\frac{du_1}{dx} + (\alpha^2 - K^2)u_1 = 0 \longrightarrow (1)$$

$$\frac{d^2u_2}{dx^2} + 2iK\frac{du_2}{dx} - (\beta^2 + K^2)u_2 = 0. \longrightarrow (2)$$

Solving both we get

$$u_1(x) = Ae^{i(\alpha-k)x} + Be^{-i(\alpha+k)x} \longrightarrow (3)$$

$$u_2(x) = Ce^{(\beta-ik)x} + De^{-(\beta+ik)x} \longrightarrow (4)$$

Applying boundary conditions of

$u_1(x=0) = u_2(x=0)$ & $u_1(x=a) = u_2(x=-b)$

$\frac{du_1}{dx}(x=0) = \frac{du_2}{dx}(x=0)$ $\frac{du_1}{dx}(x=a) = \frac{du_2}{dx}(x=-b)$

and solving with determinant method

finally we get;

$$\boxed{\frac{\alpha^2+\beta^2}{2\alpha\beta}\sinh(\beta b)\sin(\alpha a) + \cosh(\beta b)\cos(\alpha a) = \cos K(a+b)}$$

For $V_0 \to \infty$ and $b \to 0$ ($V_0 b$ = constant)

$\alpha^2 + \beta^2 = \frac{2mV_0}{\hbar^2}$, $\sinh\beta b \to \beta b$ and $\cosh\beta b \to 1$.

So eq^n - becomes:

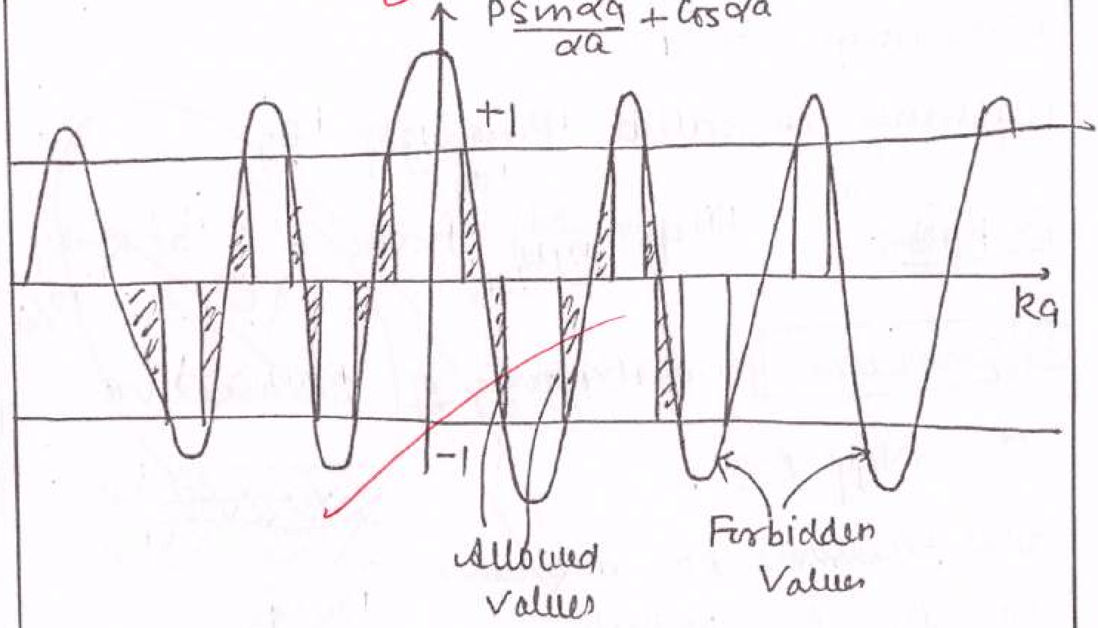
$$\frac{mV_0\,ab}{\alpha a}\sin\alpha a + \cos\alpha a = \cos Ka$$

or $\boxed{P\frac{\sin \alpha a}{\alpha a} + \cos \alpha a = \cos ka}$

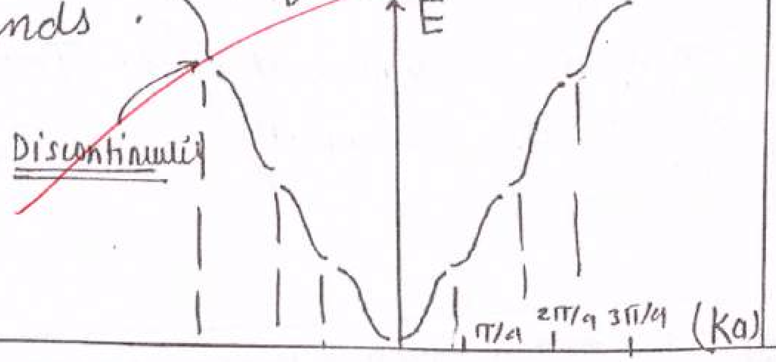
with $P = \frac{mV_0 ab}{\hbar^2}$; area of $V_0 b$ plot.

The equation shows that for $\boxed{1 \leq \cos ka \leq 1}$

there must be some allowed values only.

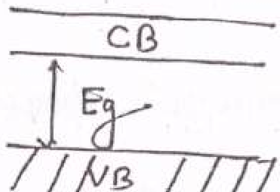


Hence there are only some values of energy are allowed and others are forbidden which leads to the formation of energy bands.



Band structure leads to: Separation of materials:

① (Insulators) – conduction band is empty while valence band is full. It requires energy for e^- to jump to CB.



Difference is called Band gap Eg.

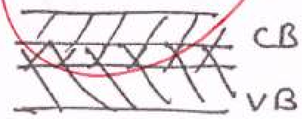
Ex: Rubber or $N_{eff} = \frac{2}{(\pi/4)} \int_{-K_1}^{K_1} f_k \, dk = 0$ at $K = K_1 = \pi/a$.

10/15

② Conductor – overlapping of CB and VB.

or $N_{eff} \neq 0$.

free electrons are available in CB. Ex: Metals

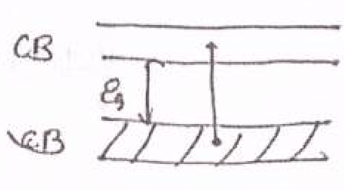


No Eg.

③ Semiconductor – Eg less than Insulators but no overlap. If Temp is risen than few electrons in C.B.

Intrinsic semiconductors at T=0K are essentially insulator

Ex: Ge, Si.



Ans 3(c)

SPECIFIC HEAT
~~Entropy~~ of Superconductors:

$$C_v = \gamma T + \beta T^3 \quad \text{(at Low temp.)}$$

$$\downarrow$$

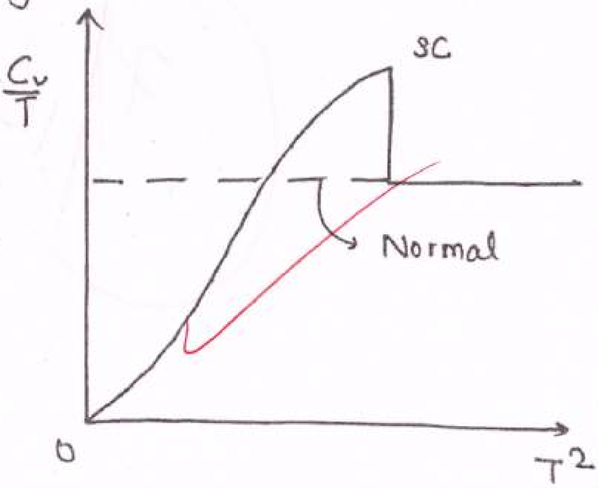
$$C_v = C_e + C_L$$

C_e → electronic

C_L → Lattice

In Superconductors (S.C.) Lattice C_L is same but C_e varies differently hence overall C_v behaves differently than normal metal.

$\frac{C_v}{T}$ vs T^2 is linear for normal conductor but for superconductors it varies.



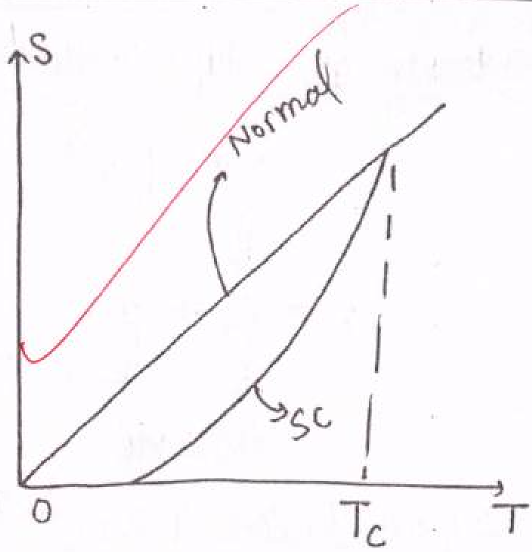
$C_{es} \propto e^{-\Delta/K_BT}$. Δ = Energy gap.

It requires for superconducting e^- to raise to excited level hence energy is required.

So the E_g comes into being.

(ii) $\boxed{\text{Entropy}}$

In superconducting state entropy goes down rapidly than the normal metal.



There are cooper-pairs which behave as bosons in SC.

→ SC state is ~~more~~ ordered.

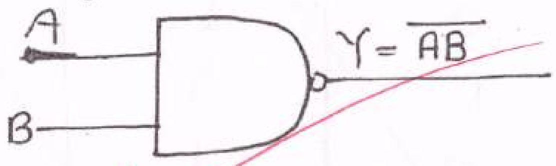
7/10

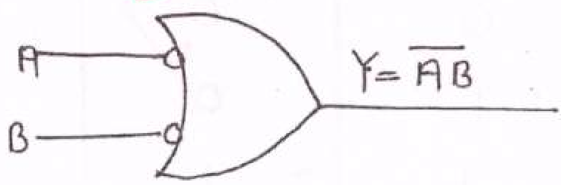
Ans 3(d) NAND and NOR gates are universal gates because any other gate can be realised using these two gates individually.

Why they are universal gates ?

NAND gate

① Logic diagram –





Bubbled OR = NAND

② Boolean equation : A NAND B = $\overline{AB}$

③ Truth Table :

A	B	$Y = \overline{AB}$
0	0	1
0	1	1
1	0	1
1	1	0

3/10

Question - 5

Ans 5(a) Full adder circuit — adds 3 bits of inputs i.e. A, B, C'.

Truth Table

A	B	C'	Sum $S = A + B + C'$	Carry C
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	1	0	0	1
1	D	1	0	1
1	1	1	1	1

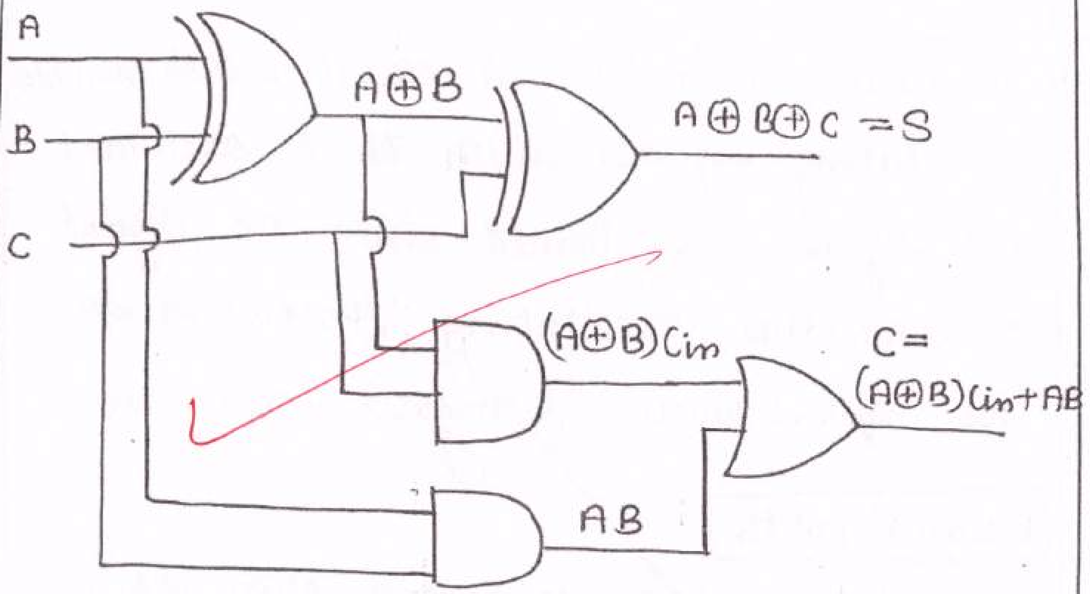
In above expressions of truth table, logic becomes

$\boxed{S = A \oplus B \oplus C'}$

while for carry $\boxed{C = (A \oplus B)C' + AB}$.

where $A \oplus B = A\bar{B} + B\bar{A}$ called X-OR.

Circuit for full adder



Full Adder Circuit

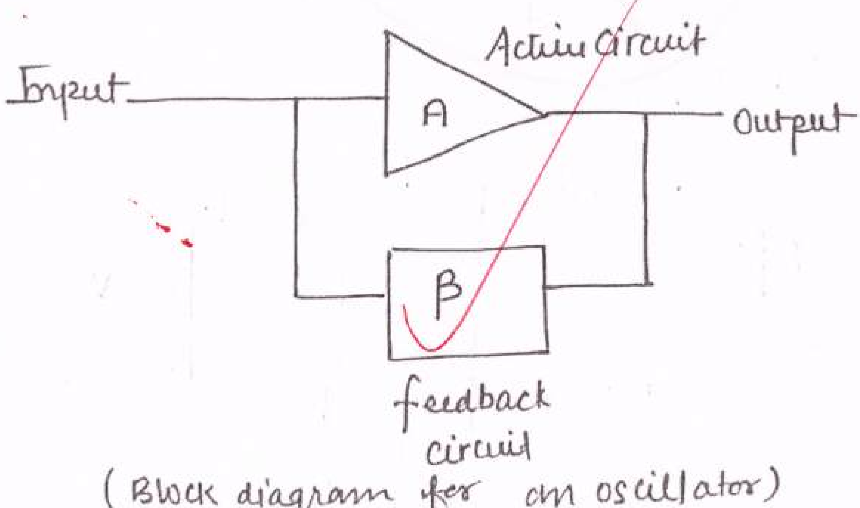
9/15

Ans5(b) Oscillator circuit - A circuit which provides oscillating signals with time such that dc signal is turned into ac signal or some other oscillating functions such as square wave, triangular waves etc.

Essential parts

(1) Oscillation creating components such as tank circuit.

(2) Feedback Loop for sustaining the oscillations.



(Block diagram for an oscillator)

(3) Barkhausen's criteria of combined gain Aβ must be unity.

Hartley oscillator:

Frequency $f = \frac{1}{2\pi\sqrt{LC}}$

to designe for $f = 1600$ Hz.

$$LC = \frac{1}{4\pi^2 f^2} = \frac{1}{4\pi^2 \times (1600)^2}$$

$$= 9.9 \times 10^{-9}$$

$$LC \sim 10 \times 10^{-9}$$

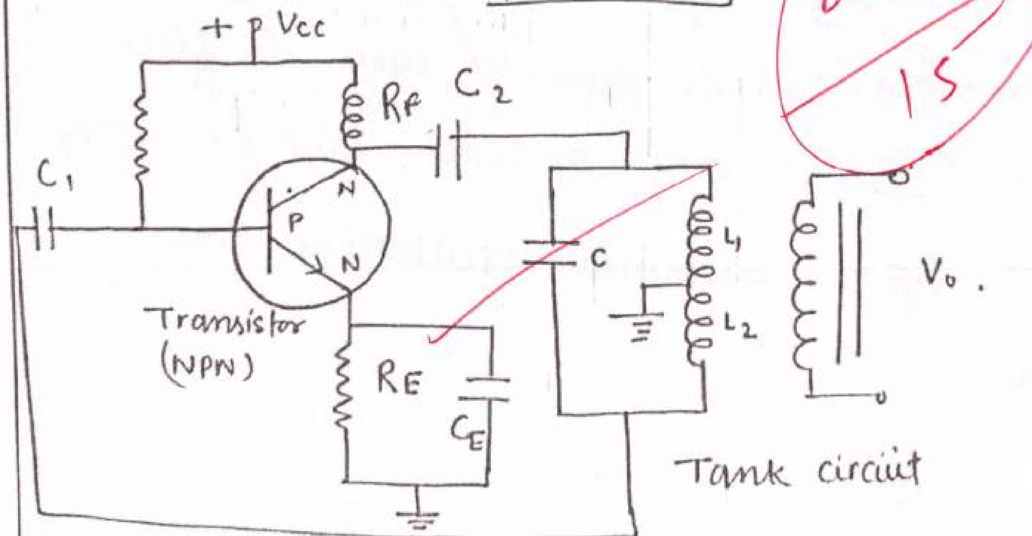
Let $C = 0.1\mu F = 10^{-7} F$

So $L = \frac{10 \times 10^{-9}}{10^{-7}} = 10 \times 10^{-2} = 0.1$ Henry

Hence $\boxed{L = 0.1\ H}$ $\boxed{C = 0.1\mu F}$

$8\frac{1}{2} + \frac{1}{2} = 9$

15



(Hartley oscillator)

Explaination :

The active component is NPN transistor which creates a phase shift of 180° in signal and the tank circuit creates oscillating signal with phase of 180°.
Hence total phase becomes 360° (No change)
The tank circuit is connected back to the input as feedback which drives the oscillations sustainably.
The capacitor C_2 blocks any dc current from entering the tank circuit.
The choke coil R_F permits easy dc flow but CHOKES any ac with high frequency.
for $A\beta=1$, sustained oscillations are got.

Ans 5(c) Advantages of MOSFET over JFET:

1. No Gate current due to isolation by SiO_2.
2. Smaller than JFET leading to incorporation in smaller Integrated circuits (ICs)
3. Very small noise
4. Very high input impedence than JFET and hence very useful in CASCADING amplifiers
5. Two modes to work with — enhancement and depletion. So a range of applications
6. More temperature stable than JFET.
7. works with good range of frequencies.
8. It has 4 terminals; Source, Drain, Gate, Substrate

∘ $\boxed{\text{MOSFET}}$ = Metal Oxide Semiconductor Field Effect Transistor is a UNIPOLAR Transistor with uses SiO_2 as an insulating layer between GATE and CHANNEL.

(n- channel MOSFET)

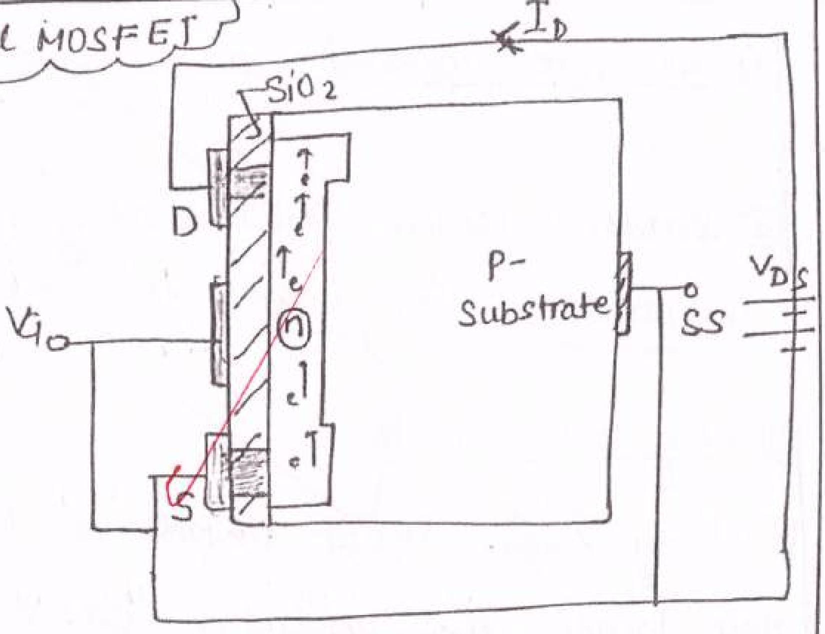


Fig. MOSFET

P-substrate made up of Silica based material is linked with n-type heavily doped channel.

→ Gate is isolated from channel-n.

→ Drain (D) and source (S) are connected through metal contact with 'n' channel.

→ Substrate 'SS' is connected to p-substrate with source.

→ Gate (G) is connected with source (S)

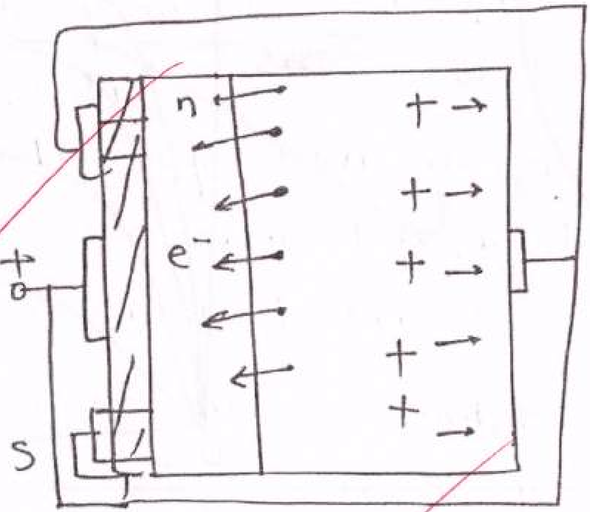
→ A battery is connected between D and S. terminals.

(Working)

→ Initially when $V_{GS} = 0V$, the current I_D flows. As we increase V_{DS} current increases and after ~~pinch off~~ certain voltage it becomes saturated I_{SS} (at $V_G = 0V$)

→ As $\boxed{V_{GS} \text{ is increased positively or}}$ V_{GS} = +ve then it attracts the electrons from substrate and repels holes in p-substrate.

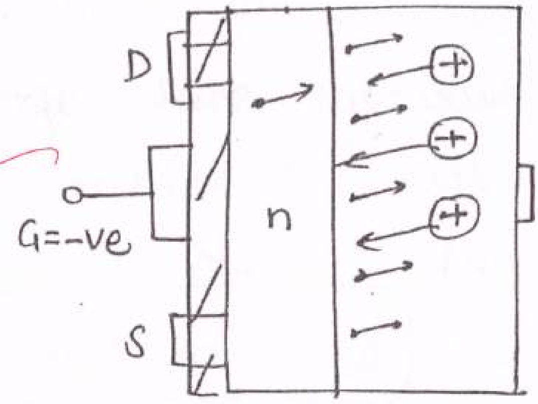
→ It enhances the current I_{DS} and it saturates at greater values.



This is known as ENHANCEMENT TYPE REGION.

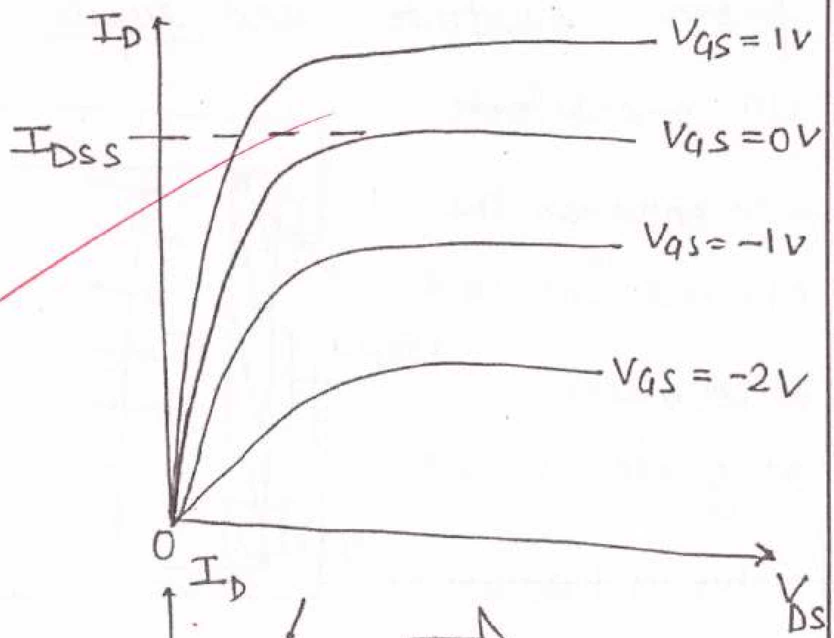
• Now if $V_{GS} < 0V$ or negative then in channel 'n', it repels electrons, attracting holes from p-substrate leading to depletion in current or decreased I_{DS}. This is known as

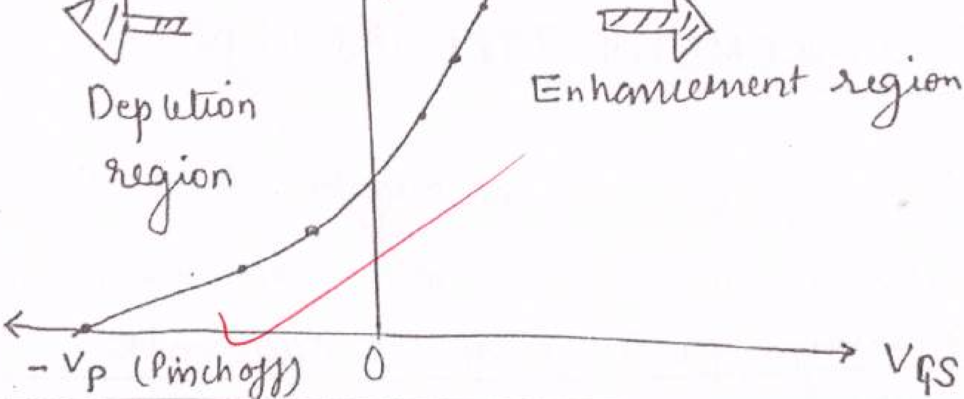
$\boxed{\text{depletion region}}$.



$\boxed{\text{Depletion type MOSFET}}$

$\boxed{I_D \text{ vs } I_{DS}}$





Ans-6

Ans 6(a)

$$Y = A(B+\overline{A}) + B(A+\overline{B})$$
$$= AB + A\cdot A + BA + B\cdot\overline{B}$$
$$= AB + A + AB + 0 \qquad (\because B\overline{B} = 0)$$
$$= AB + A \qquad (\because AB + AB = AB)$$
$$= A(B+1) \qquad (\because 1 + B = 1)$$
$$\boxed{Y = A}$$

Realisation using NAND gates.

$$Y = A = \overline{\overline{A}}$$

so

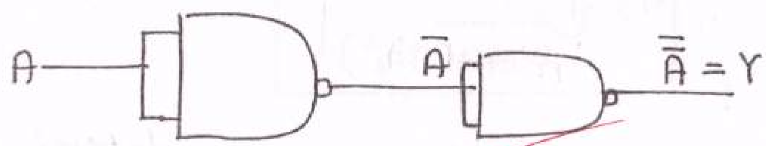


Fig. A using NAND gates

6½ / 10

Ans 6(b) Energy $E(\vec{k}) = Ak^2 + Bk^4$.

Effective mass $m_e^* = \left(\frac{1}{\hbar^2}\frac{d^2E}{dk^2}\right)^{-1}$

$$\frac{d}{dk}\left(\frac{dE}{dk}\right) = \frac{d}{dk}\left(2Ak + 4Bk^3\right)$$

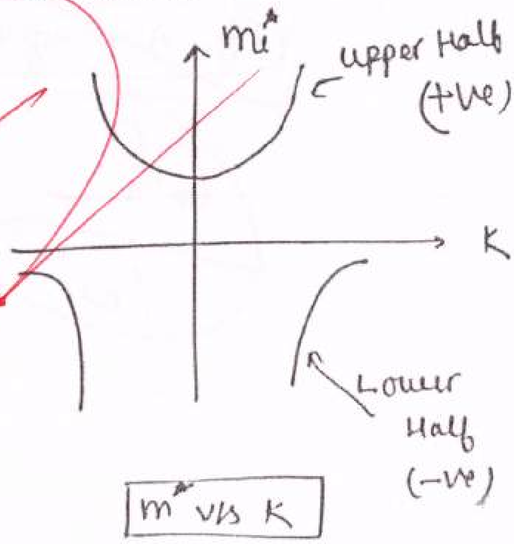
$$= 2A + 12Bk^2$$

at $k = k_0$ $\frac{d^2E}{dk^2} = 2A + 12Bk_0^2$.

so $m_e^* = \frac{\hbar^2}{2A + 12Bk_0^2} = \frac{\hbar^2}{2}\cdot\left(\frac{1}{A + 6Bk_0^2}\right)$

$$\boxed{m_e^* = \frac{\hbar^2}{2(A + 6Bk_0^2)}}$$

Effective mass of e^- is mass in exteral electric field.



6½

12

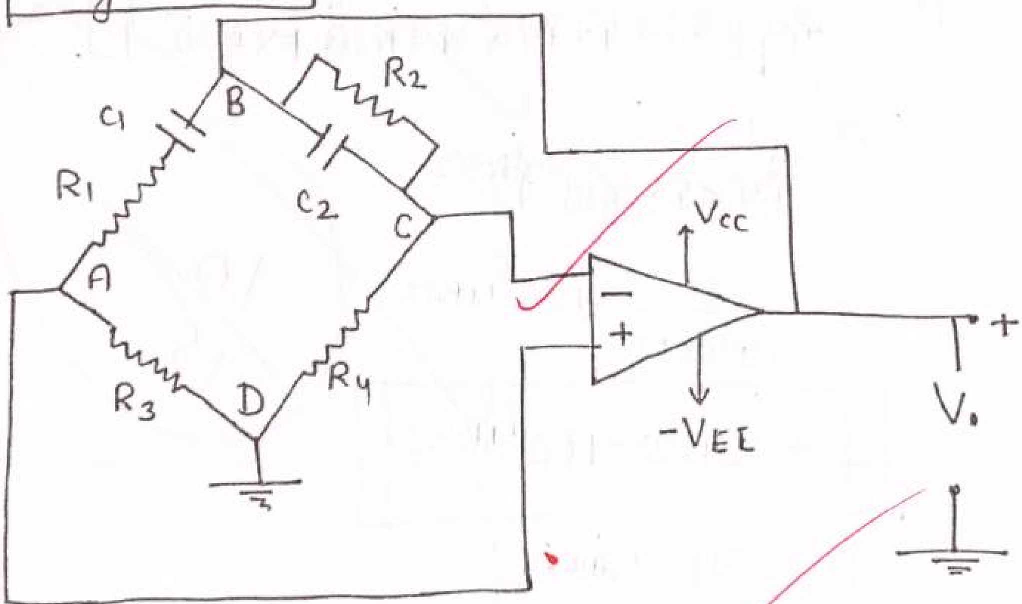
Ans6(c) Wein Bridge Oscillator – It utilizes the wein Bridge consisting of a circuit with capacitors and resistors and in a particular configuration.

→ It generates oscillating signals using –

① Transistors

② OPAMPs as active components.

Using OPAMP



Connecting the D point to ground and feeding −ve of OPAMP with C-point & +ve with A point. Feedback is used with B-point.

Now, with this arrangement oscillations are produced with frequency-

$$\boxed{f = \frac{1}{2\pi\sqrt{R_1R_2C_1C_2}}.}$$

Given: $R_1 = R_2 = R_3 = R_4 = 51\ k\Omega = 51\times10^3\ \Omega$

$C_1 = C_2 = 0.001\mu F = 1\times10^{-9}\ F$

$$f = \frac{1}{2\pi\left[51\times51\times10^6\times1\times10^{-9}\times1\times10^{-9}\right]^{1/2}}.$$

$$= \frac{1}{2\pi\times51\times(10^{-12})^{1/2}}$$

$$= \frac{1}{2\pi\times51}\times10^6\ Hz$$

$$\boxed{f = 3.12\times10^3\ Hz}$$

$$\boxed{f = 3.12\ KHz}.$$

10/15

Ans6(d) Advantages of using negative feedback in an amplifier —

① Negative feedback STABELIZES the gain of the oscillator.

② It increases its BANDWIDTH.

③ It decreases the linear or HARMONIC DISTORTIONS in a circuit.

Though it has drawback that it changes phase by 180° and doesn't AID the input as is ~~does~~ done by positive feedback.

2

(ii) Given: RC coupled amplifier has frequency

gain = 200

frequency response = 100 Hz to 100 kHz

Negative feedback $\beta = 0.02$.

Parameter of performance = ?